Section 9.5 - Inverse Functions

Focus - $f(x) = 2x - 1$

$g(x) = x^2 + 2$

Find:

1) $(f \circ g \circ f)(2) = f(g(f(2))) \Rightarrow$

$f(2) = 2(2) - 1 = 3$
$g(3) = 3^2 + 2 = 11$
$f(11) = 2(11) - 1 = \boxed{21}$

2) $g(g(f(f(1)))) \Rightarrow$

$f(1) = 2(1) - 1 = 1$
$f(1) = 2(1) - 1 = 1$
$g(1) = 1^2 + 2 = 3$
$g(3) = 3^2 + 2 = \boxed{11}$

3) $f(g(x)) = f(x^2+2)$
$= 2(x^2+2) - 1$
$= 2x^2 + 4 - 1 = \boxed{2x^2+3}$

4) $(fg)(x) = (2x-1)(x^2+2)$
$= 2x^3 + 4x - x^2 - 2$
$= \boxed{2x^3 - x^2 + 4x - 2}$

Inverse Functions

RECAP → $y = \frac{2x+5}{4}$ Find the inverse function.

1) ✓ to see if function is OTO (passes VLT/HLT)

2) switch x & y $\qquad x = \frac{2y+5}{4}$

3) Solve for y. $\qquad 4x = 2y + 5$

$4x - 5 = 2y$

4) change y into y^{-1} or $f^{-1}(x)$

$y^{-1} = \frac{4x-5}{2}$

$f^{-1}(x)$

To determine if functions are inverses, <u>use compositions</u>.

* If $f(x)$ and $g(x)$ are inverses, then:

$$f(g(x)) = x$$

<u>AND</u>

$$g(f(x)) = x$$

<u>ex</u>: $f(x) = 3x - 1$, $g(x) = \frac{1}{3}x + 1$

$$f(g(x)) = f\left(\tfrac{1}{3}x + 1\right)$$
$$= 3\left(\tfrac{1}{3}x + 1\right) - 1$$
$$= x + 3 - 1$$
$$= x - 2$$
$$\neq x$$

$\therefore$ $f(x)$ and $g(x)$ are <u>NOT</u> inverses!

<u>ex</u>: $f(x) = \frac{1}{x-1}$, $g(x) = \frac{1}{x} + 1$ $(x \neq 0, 1)$

$$f(g(x)) = f\left(\tfrac{1}{x} + 1\right)$$
$$= \frac{1}{\frac{1}{x} + 1 - 1} = \frac{1}{\frac{1}{x} + 0} = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = \boxed{x} \checkmark$$

$$g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1}} + 1 = \left(1 \cdot \frac{x-1}{1}\right) + 1$$
$$= x - 1 + 1$$
$$= \boxed{x} \checkmark$$

$\therefore$ $f(x)$ and $g(x)$ are inverses!

$\underline{\underline{ex}}$: $f(x) = \frac{1}{9}x^2$ if $x \geq 0$, $g(x) = 3\sqrt{x}$

⋆ must be OTO!!

$$f(g(x)) = f(3\sqrt{x}) = \frac{1}{9}(3\sqrt{x})^2$$
$$= \frac{1}{9}(9x)$$
$$= \boxed{x} \checkmark$$

$\therefore$ f(x) and g(x) are inverses!

$$g(f(x)) = g\left(\frac{1}{9}x^2\right) = 3\sqrt{\frac{1}{9}x^2}$$
$$= 3\left(\frac{1}{3}\right)|x| = |x|$$
$$= \boxed{x} \checkmark$$

since $x \geq 0$, take $+x$!!

$\underline{\underline{ex}}$: $f(x) = x^2 + 5$, $g(x) = \sqrt{x} - 5$
$(x \geq 0)$

$$f(g(x)) = f(\sqrt{x} - 5) = (\sqrt{x} - 5)^2 + 5$$
$$= x - 10\sqrt{x} + 25 + 5$$
$$= x - 10\sqrt{x} + 30$$
$$\neq x$$

$$g(f(x)) = g(x^2 + 5) = \sqrt{x^2 + 5} - 5$$
$$\neq x$$

$\therefore$ f(x) and g(x) are NOT inverses!

REMINDERS →

1) NOT every function has an inverse!
2) If (a,b) is on f(x), then (b,a) is on $f^{-1}(x)$!
3) Functions are inverses if they are reflections over $y = x$!